

## The instantaneous Earth rotation – still inaccessible?



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### Abstract

Nowadays, space geodesy, such as Very Long Baseline Interferometry (VLBI) and Global Navigation Satellite System (GNSS) and Satellite Laser Ranging (SLR), allows determining Earth orientation to a fraction of 1 milliarsecond with daily to hourly resolution. This should not prevent us from studying other innovative and powerful technologies. A new emerging technology, called ring laser gyroscope, is a high-precision tool that provides us with extra information in the daily and sub-daily time domain. The experimental determination of the amplitudes of the forced diurnal polar motion is for example exclusively allocated to the ring laser technique. Another aspect, in which ring lasers could emphasize their supremacy, is the determination of the motion of the Earth-fixed frame w.r.t. the instantaneous Earth rotation axis. However, present ring lasers are huge constructions extremely sensitive to external effects, e.g., temperature variations.

This paper illuminates the relationship between various Earth rotation axes, in the large sense, and discusses the separation between polar motion and nutation of these axes in space and w.r.t. the Earth body. The second part covers the expected benefit of ring laser observables.

### Kurzfassung

Die geodätischen Weltraumtechniken, wie VLBI, GNSS und SLR, erlauben heute die Bestimmung der Orientierung einer mittleren Polachse der Erde im Raum und relativ zum Erdkörper mit einer Genauigkeit einiger Zehntel Millibogensekunden und einer zeitlichen Auflösung im Stundenbereich. Dieser offensichtliche Erfolg sollte uns aber nicht daran hindern, auch neue innovative Techniken zur Bestimmung der Erdrotation, wie z.B. Ringlaser, näher zu untersuchen. Ringlaser sind hochpräzise Instrumente, die uns speziell bei subtäglicher Auflösung erlauben, die Bewegung der festen Erdkruste relativ zur wahren Rotationsachse zu verfolgen. Es muss allerdings bemerkt werden, dass die Ringlaser große bauliche Anlagen erfordern und äußerst sensibel auf Einflüsse wie Temperaturänderung etc. reagieren.

Der vorliegende Artikel beleuchtet den Zusammenhang der unterschiedlichen Erddrehachsen und diskutiert demgemäss die in Polbewegung und Nutation aufgespaltenen Bewegungen dieser Achsen im Raum und relativ zum Erdkörper. Speziell werden dann im zweiten Teil die neuen von den Ringlasern zu erwartenden Messgrößen behandelt.

### 1. Fundamentals

Generally, two fundamental frames of reference are used for studies of Earth rotation: the quasi-inertial celestial coordinate system tightened to the directions of extragalactic sources, and the terrestrial coordinate system attached to several observatories, perpetually in motion, located on the Earth surface [20]. The first realization holds the acronym ICRF (International Celestial Reference Frame), whilst the second is called ITRF (International Terrestrial Reference Frame) [1]. The study of Earth rotation, in its basics, is already rather complicated. But, in the case of the Earth composed of an atmosphere, oceans, crust, mantle, liquid outer and solid inner core [11, 13, 15], things turn out to get extremely intriguing. In that case, the concept of Earth rotation, as a whole, is deprived of its initial sense. Which axis of rotation is meant, when the terminology “Earth rotation” appears in literature, and what are the

underlying reference frames? In theoretical derivations, the perturbed instantaneous rotation vector is usually described in the “Tisserand mean-mantle” frame. In this frame, the relative angular momentum of the crust and mantle vanishes, while the one of the atmosphere, oceans and core does not. Another frame, introduced by Darwin G.H. [10], is chosen such that the diagonal components of the Earth inertia tensor keep constant over time [16]. Such frames considerably simplify the theoretical study of geophysical influences on Earth rotation, but how are such frames realized? Do such frames allow for a convenient comparison of excitation functions to astronomical and geodetic observations?

For some decades, space geodetic techniques were the primary means to monitor Earth rotation variations [2, 23, 26, 32]. Let us first recapitulate some basic notions relevant for discussing Earth rotation displayed in Figure 1.

If we set external torques (due to the presence of external bodies of our solar system) acting on the Earth equal to zero, the angular momentum vector  $\vec{L}$  remains constant over time w.r.t. an inertial frame. Hence we assume that the Earth system, i.e., without the Moon, is a closed and conservative system. Under this hypothesis, the conservation of angular momentum admits an exchange between the Earth inertia tensor  $\Theta$ , the rotation vector  $\vec{\Omega}$  and the relative angular momentum  $\vec{H}$ . Figure 1 shows the relationship between these three vectors in the figure axis system  $(\vec{x}, \vec{z})$  [19]. The  $\vec{z}$ -component is directed towards the Earth greatest moment of inertia, while the  $\vec{x}$ -component is pointed towards the Earth smallest moment of inertia. The direction of the pole of the Earth-fixed frame is denoted by  $\vec{P}_{ITRF}$ , i.e., the pole of the most recent ITRF. If the length of day (LOD) increases, hypothesizing a constant inertia tensor, the relative angular momentum increases too. Fortunately, the inertia tensor absorbs most of the surplus angular momentum, leaving relative angular momentum quasi unaffected. Space geodetic techniques usually observe an intermediate axis, different from the Earth rotation axis with a time resolution of one day and even shorter.

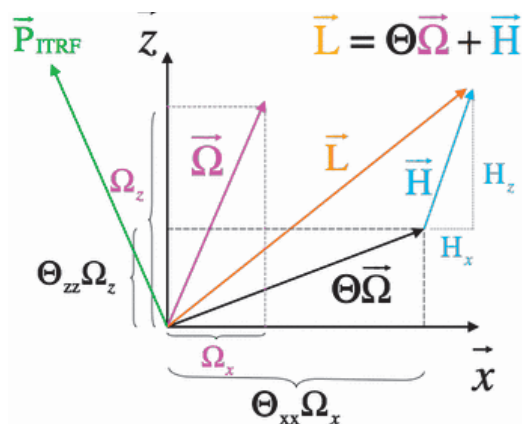


Figure 1: Relationship between the Earth angular momentum  $\vec{L}$ , the inertia tensor  $\Theta$ , the rotation vector  $\vec{\Omega}$  and the relative angular momentum  $\vec{H}$  w.r.t. the principal unit axes of inertia  $\vec{x}$  and  $\vec{z}$ .

Recently, the emerging ring laser technology, making use of the Sagnac effect, developed rapidly and provides the instantaneous Earth rotation vector without looking outside the Earth [8, 25]. The Sagnac effect is obtained by the frequency difference between counter-rotating beams, when the system or Earth rotates.

## 2. The motion of the Earth rotation axis

The motion of the rotation axis of an elastic Earth in space has been first modeled by W. Schweydar in 1917 [27]. Since then, Earth models evolved, and this motion has been modeled semi-analytically (IAU 2000) to an accuracy of less than  $200 \mu\text{as}$  [17, 31, 6]. Recently, Krasinsky developed this motion to near observational perfection groping the  $100 \mu\text{as}$  accuracy level (ca. 3 mm on Earth surface), for daily values [14]. Figure 2 displays the most relevant poles of reference to Earth rotation and will be essential in the understanding of subsection 2.1. These poles are: the pole of the ICRF, the initial Celestial Intermediate Pole CIP ( $\text{CIP}_0$ ), the final CIP, and the pole of the ITRF. An unsolved task is to predict the remaining offsets between the celestial pole  $\text{CIP}_0$  and the CIP. The Earth rotation pole is located close to the CIP: its formulation w.r.t. the  $\text{CIP}_0$  will be presented in subsection 2.1. On the other hand, the motion of the non-rigid Earth w.r.t. the Earth rotation axis, is still unpredictable at the 200 milliarcsecond (mas) level [12, 34], and is therefore perpetually monitored. Annual meteorological effects drive the yearly signal of this motion, while the prediction of the free component of polar motion, called Chandler Wobble (CW), remains a topic under investigation since more than one century. In recent literature, the description “free nutation” is reserved for other resonant modes such as the Free Core Nutation (FCN). This terminology is also consistent with the definitions provided by the IERS Conventions 2003 (see Figure 4). Discussions and papers on the multiple-peaks spectrum of the CW have existed in the past, and recent publications remind us of this possible feature [21]. This splitting of frequencies would at least explain the great Chandler Wobble change during the years 1930–1940. However, it should be noted that the multiple-peaks model is only one possible way of describing the time-variable CW signal. Moreover, this model is rather controversial because it is based solely on data analysis. So far, nobody could give any physical reasoning for it. The great change of the CW in the 1930's can be interpreted also within other models as shown in [5, 29]. A great recent progress in the excitation studies of this free wobble has been attained using the model in which the CW signal is described as the free oscillator with damping excited by the random processes taking place in the dynamically coupled system atmosphere-oceans [33, 13].

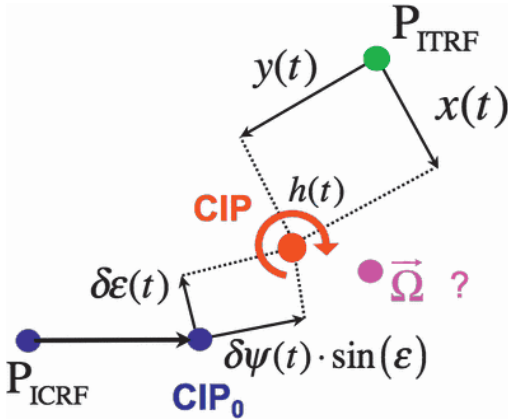


Figure 2: Representation of relevant poles of reference to Earth rotation.

### 2.1. Polar motion and universal time vs. instantaneous rotation vector

Reminding Figure 2, the rotation matrix  $B(t)$ , which in our restricted case accounts only for polar motion,  $x(t)$  and  $y(t)$ , and the hour angle of the true equinox of date  $h(t)$ , transforms vectors from a terrestrial (TRS) to an intermediate reference system [12], i.e., the CIP:

$$X(t) = \begin{pmatrix} \cos[x(t)] & 0 & -\sin[x(t)] \\ 0 & 1 & 0 \\ \sin[x(t)] & 0 & \cos[x(t)] \end{pmatrix} \quad (1)$$

$$Y(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[y(t)] & \sin[y(t)] \\ 0 & -\sin[y(t)] & \cos[y(t)] \end{pmatrix} \quad (2)$$

$$h(t) = \int \Omega_0 \cdot (1 + z(t)) dt \quad (3)$$

$$S(t) = \begin{pmatrix} \cos[h(t)] & -\sin[h(t)] & 0 \\ \sin[h(t)] & \cos[h(t)] & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$$B(t) = S(t) \cdot X(t) \cdot Y(t) \quad (5)$$

The skew-symmetric tensor  $\Omega(t)$  allows recovering the instantaneous Earth rotation vector w.r.t. the TRS:

$$\Omega(t) = B^T(t) \cdot \delta_t B(t) = \begin{pmatrix} 0 & \Omega_z(t) & -\Omega_y(t) \\ -\Omega_z(t) & 0 & \Omega_x(t) \\ \Omega_y(t) & -\Omega_x(t) & 0 \end{pmatrix} \quad (6)$$

In deriving the instantaneous Earth rotation vector, it is assumed that the precession/nutation model is perfectly known [6]. The instantaneous Earth rotation vector, which is considered in the

dynamical theories and observed by the ring laser, concerns Earth rotation relative to the inertial space. In equation (7), the spatial reference is an intermediate reference system, which moves in inertial space (transition from  $P_{ICRF}$  to  $CIP_0$ , then to CIP, as shown in Figure 2) causing additional contributions to  $\tilde{\Omega}(t)$ . A detailed discussion of this problem is given in [4].

The traditional way for obtaining the Earth rotation vector keeps only terms to first order in small quantities, i.e., polar motion  $x(t)$  and  $y(t)$  and departure  $z(t)$  from uniform spin at the mean sidereal rotation speed  $\Omega_0$  of the Earth [4, 34, 12]. Let us consider the non-linearized skew-symmetric tensor  $\Omega(t)$  and give a complete geometrical interpretation of the motion of the Earth rotation vector w.r.t. the normalized vector  $\vec{r}_{CIP_0}$  of the  $CIP_0$ :

$$\tilde{\Omega}(t) = \begin{pmatrix} \Omega_x(t) \\ \Omega_y(t) \\ \Omega_z(t) \end{pmatrix} = \begin{pmatrix} \Omega_0 \sin[x(t)] + z(t) \cdot \Omega_0 \sin[x(t)] - \partial_t y(t) \\ -\Omega_0 \cos[x(t)] \cdot \sin[y(t)] - \Omega_0 \cdot z(t) \cdot \cos[x(t)] \cdot \sin[y(t)] - \cos[y(t)] \cdot \partial_t x(t) \\ \Omega_0 \cos[x(t)] \cdot \cos[y(t)] + \Omega_0 \cdot z(t) \cdot \cos[x(t)] \cdot \cos[y(t)] - \sin[y(t)] \cdot \partial_t x(t) \end{pmatrix} \quad (7)$$

which can be synthesized into:

$$\begin{pmatrix} \Omega_z(t) \\ -\Omega_y(t) \\ \Omega_x(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \cdot \partial_t y(t) + \partial_t h(t) \cdot \vec{r}_{CIP_0} + \partial_{y(t)} \vec{r}_{CIP_0} \cdot \partial_t x(t) \quad (8)$$

Formula (8) relates the Earth rotation vector to the initial CIP, i.e. the  $CIP_0$  (see equation 10). Its magnitude reads:

$$|\tilde{\Omega}(t)| = \left\{ \Omega_0^2 + \Omega_0^2 \cdot z(t)^2 + [\partial_t x(t)]^2 - 2\Omega_0 \sin[x(t)] \cdot \partial_t y(t) + \dots \right\}^{\frac{1}{2}} + \left\{ [\partial_t y(t)]^2 + 2\Omega_0 \cdot z(t) \cdot [\Omega_0 - \sin[x(t)] \cdot \partial_t y(t)] \right\} \quad (9)$$

To first approximation, the initial unit CIP vector, and its partial derivative w.r.t. polar motion in the y-component, follows as:

$$\vec{r}_{CIP_0} = \begin{pmatrix} \cos[x(t)] \cdot \cos[y(t)] \\ \cos[x(t)] \cdot \sin[y(t)] \\ \sin[x(t)] \end{pmatrix} \approx \begin{pmatrix} 1 \\ y(t) \\ x(t) \end{pmatrix} \quad (10)$$

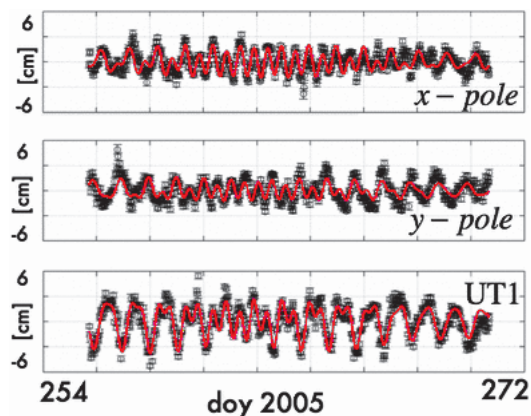
$$\partial_{y(t)} \vec{r}_{CIP_0} = \begin{pmatrix} -\sin[y(t)] \\ \cos[y(t)] \\ 0 \end{pmatrix} \approx \begin{pmatrix} -y(t) \\ 1 \\ 0 \end{pmatrix} \quad (11)$$

Keeping only terms to first order in small quantities reduces the Earth rotation vector and its magnitude to the familiar form [4, 12]:

$$\begin{pmatrix} \Omega_z(t) \\ -\Omega_y(t) \\ \Omega_x(t) \end{pmatrix} \approx \begin{pmatrix} \Omega_0 \cdot [1 + z(t)] \\ \Omega_0 \cdot y(t) + \partial_t x(t) \\ \Omega_0 \cdot x(t) - \partial_t y(t) \end{pmatrix} \quad (12)$$

$$|\vec{\Omega}(t)| \approx \Omega_0 \cdot (1 + z(t)) \quad (13)$$

We would like to make clear that present polar motion and universal time estimates, in fact, do contain much more information than only the net-rotation (NNR) of the station coordinate corrections. The reason is that more than 70% of Earth surface is covered by the oceans where no geodetic sites are established. All signals in the oceans, which do not significantly affect station coordinates, do have their signatures in the Earth rotation parameters.



**Figure 3:** De-trended hourly polar motion and UT1 variations from VLBI (black circles) and results by using a source code of the ocean tidal model worked out by Ray et al. (1994), extended by Chao et al. (1996) and interpolated by Eanes for smaller tidal constituents (red line) during the VLBI CONT05 campaign. Variations are projected to the Earth surface and given in units of [cm].

Figure 3 shows de-trended hourly polar motion and universal time offsets (a smooth low-frequency function has been removed) during the VLBI CONT05 campaign. The VLBI software package OCCAM61E has been used [28]. The difference between the black circles and the red line, which is predicted from the most recent ocean tidal model adapted by Eanes [3], is in median less than 6 mm for each component. In fact, the tables provided by the IERS Conventions 2003 are based on the ocean tidal model worked out by [22] and extended by [9] by computing additionally influence on polar motion. Eanes

interpolated this model for smaller tidal constituents in a standard way, using the tidal potential and the admittance function [3].

## 2.2. Polar motion versus nutation

The basic and clear distinction between polar motion and precession/nutation started getting more complicated since space geodetic measurements allowed for subdaily temporal resolution of Earth rotation parameters, i.e., polar motion and universal time or length of day (LOD).

For interested readers in historical aspects, discussing the rather confusing terminology in the literature from different points of views, we refer to more extensive studies, e.g., [7, 18, 35].

Till recently, space geodetic techniques remained the only practical means of monitoring Earth rotation variations. However, space geodesy needs to attach their observations to external objects, e.g., quasars, stars, planets, satellites, and are therefore principally unable to record clean geophysical polar motion without aliasing from precession/nutation [8].

Space geodetic techniques observe only the time dependent transformation matrix  $B(t)$  (including precession and nutation) between the ITRF and the ICRF. The components of the instantaneous rotation vector are not needed to define this matrix. Moreover, the dynamical theories of Earth rotation can also be expressed in terms of the components of the matrix  $B(t)$ . This matrix has three degrees of freedom, and hence can be described by three mutually independent parameters. For various reasons, e.g., tradition, continuity of software, etc., the scientific community decided to keep the convention of using five parameters (two for polar motion, two for precession-nutation and one for spin). Therefore it became necessary to introduce an additional constraint (see Figure 4) to remove the redundancy of the parameters. This artificial problem and its solution correspond to the title of section.

The solution of this artificial problem was conventionally adopted by the scientific community and is anchored in the International Earth rotation and Reference systems Service (IERS) Conventions 2003, which define polar motion and nutation as depending on the frequency of the motion [24]. Here, every motion in the terrestrial reference frame with a retrograde period from 16 to 48 sidereal hours is attributed to nutation; all other pro- and retrograde sidereal periods are considered to pertain to polar motion (see

Figure 4). Retrograde means the opposite direction to Earth rotation. The retrograde nearly diurnal ocean tidal terms as well as terms excited by the atmosphere are considered to be part of nutation. From the astronomical point of view, this is perfectly correct, and represents one additional part of forced nutation (not externally, but internal to the Earth system).

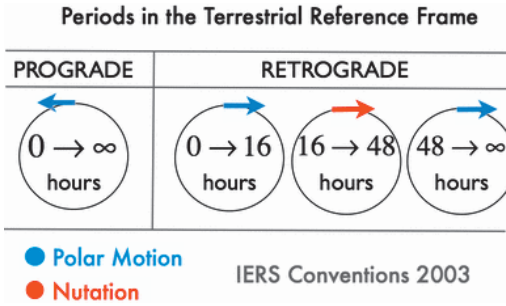


Figure 4: Definition of polar motion and nutation following the IERS Conventions 2003.

The ring laser gyroscope observes a completely different quantity, i.e., the instantaneous rotation vector, which according to equation (6) is obtained from the product of the transposed matrix  $B(t)$  and its time derivative. Each terrestrial motion of this vector, polar motion, is associated by the celestial motion, nutation. Another difference is that the observations of space geodesy are more or less limited in time resolution while the ring laser gyroscope can (at least potentially) perform continuous recordings.

Nevertheless, space geodetic techniques do detect many signals in polar motion and length of day down to the cm-level. The current performance of ring laser gyroscopes seems also being able to detect other effects beyond the Oppolzer terms (see section 3, Figure 5). This statement is also applicable to length of day variations (Figure 6). The potential of ring laser data remains still to be exploited. Recent data sets look very promising.

### 3. The ring laser gyroscope

In the last decade, the forced diurnal polar motion felt nearly into oblivion, since space geodetic techniques take this signal into account as part of precession/nutation models, e.g., IAU 2000. But recently a revival in modern Sagnac instruments, especially in ring laser gyroscopes emerged. These sophisticated instruments are sensitive to absolute rotation. Recent papers report of ring laser gyroscopes being the first tools to detect forced diurnal polar motion (arising due to lunisolar torques), first predicted by Ritter T. von Oppolzer (see Figure 7), already in the nineteenth century [30]. In reality, present single oriented circuits record the angular variation formed by the instantaneous Earth rotation vector with the area vector normal to the circuit, and are therefore sensitive to the frequency signature of the forced diurnal polar motion.

The gyrometric phase shift of a ring laser measurement is proportional to  $Q(t)$ :

$$Q(t) = A(t) \cdot \vec{n}_0(t) \cdot \vec{\Omega}(t) \tag{14}$$

where  $A(t)$  is the area formed by the beam circuit,  $\vec{n}_0(t)$  is its unit normal vector, and  $\vec{\Omega}(t)$  is the Earth rotation vector. Any variation in the angular speed or the magnitude of the Earth rotation vector translates immediately into the phase shift or  $Q(t)$ . The components of  $\vec{n}_0(t)$  must be given in the terrestrial reference frame, however, need not being highly accurate. But, changes in  $A(t)$  and  $\vec{n}_0(t)$ , due to pressure or

Phenomenon	Polar motion				
	Period [days]	Amplitude [cm]	VLBI	GNSS	Ring laser
Chandler	~432	600	✓	✓	?
Annual	~365	300	✓	✓	?
von Oppolzer	~1	60	✓*	?	✓
Ocean tides	~0.5, ~1	3	✓	✓	?

Figure 5: Sensitivity of space geodetic techniques and ring lasers to polar motion signals. Question marks denote that those techniques are not yet able to sense these signals but will probably do in future. (\* is included in nutation).

Phenomenon	Length of Day				
	Period [days]	Amplitude [cm]	VLBI	GNSS	Ring laser
Solid Earth tides	13.7, 182, ...	50	✓	✓	(✓)
Atmosphere (winds, pressure)	10-60	50	✓	✓	(✓)
Ocean tides	~0.5, ~1	3	✓	✓	?

Figure 6: Sensitivity of space geodetic techniques and ring lasers to length of day signals. Question marks denote that those techniques are not yet able to sense these signals but will probably do in future.

temperature changes are critical, given the high sensitivity requirements of the ring laser. For that reason the orientation of the ring laser is monitored by tiltmeters. However, the latter are not sensitive to locally induced rotational motions of the ground arising from shear stresses. Tidal deformations are such motions, but these harmonic tidal frequencies can be easily accounted for. Deformations arising from atmospheric and hydrological loading are other possible sources, however produce mainly surface normal stresses on the soil. In any case, local rotational motions, e.g., due to tremendous storms such as the European windstorm Kyrill in January 2007, should not be neglected when interpreting variations of  $Q(t)$  and thus indirectly of  $\vec{\Omega}(t)$ .



Figure 7: Prof. Theodor Ritter von Oppolzer.

In principle, three independently oriented circuits would be sufficient at one single site to determine the Earth rotation vector  $\vec{\Omega}(t)$  unambiguously if the a priori knowledge of  $A(t)$  and  $\vec{n}_0(t)$  is sufficiently accurate. In the case that  $A(t)$  is exactly known, five unknowns remain for one single site. Three sites equipped with three independently oriented circuits would be suffi-

cient to determine their respective tilts  $\vec{n}_0(t)$ , and the Earth rotation vector  $\vec{\Omega}(t)$  unambiguously.

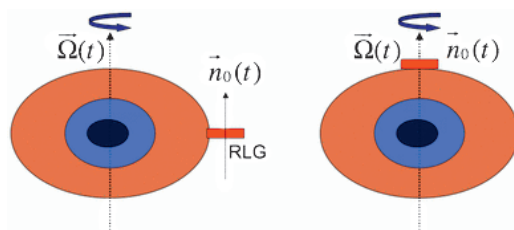


Figure 8: Equatorial (left) and polar mounting (right) of a single circuit ring laser (RLG).

At first, Figure 8 explores one further special case regarding only one single-circuit ring laser, located at the equator and having its areal unit normal vector nearly parallel to the Earth rotation vector, i.e., the angular departure of  $\vec{n}_0(t)$  from  $\vec{\Omega}(t)$ . With this mounting, the ring laser will be mainly sensitive to the change in magnitude  $\Omega$  of  $\vec{\Omega}(t)$ , i.e., to LOD variations. It will remain quasi unaffected, from both polar motion and local tilt. And secondly, in the same logic, a polar mounting with  $\vec{n}_0(t)$  parallel to  $\vec{\Omega}(t)$  senses primarily LOD variations.

#### 4. Conclusions

Very Long Baseline Interferometry (VLBI) is a geodetic technique, which enables to estimate parameters related to the shape of the Earth and orientation in inertial space. Such parameters are the difference between universal time corrected for polar motion and Universal Coordinated Time (UT1-UTC), polar motion, and nutation/precession [16]. Besides, VLBI is the most accurate technique having access to the Celestial Reference Frame (CRF) through observations of quasars. Today, polar motion and universal time observations with a temporal resolution as short as semi-hourly are obtained from VLBI, with the potential to reveal short period and episodic events with signatures below the  $100\mu\text{as}$  ( $7.5\mu\text{as}$ ) level. The Global Navigation Satellite System (GNSS) contributes effectively to nutation rates, polar motion, and length of day parameters and Satellite Laser Ranging (SLR) is also observing polar motion, LOD and geocentre variations.

Nonetheless, the ring laser provides additional information, which is not yet, and presumably in principle never, accessible through space geodetic measurements. In the future, ring lasers could become standard high-precision tools, which could measure the finest rotational phenomena of the Earth. The operation of high-

sensitivity gyroscopes (three independently oriented circuits) at several (at least three) highly stable sites would be a real breakthrough in Earth rotation science. The combination of VLBI and ring laser data at the observation level would presumably relieve one shortcoming, i.e., provide a clean physical separation between nutation offsets and polar motion for high-frequency estimates. And finally, some systematic errors of the individual monitoring techniques could potentially be identified by comparison of both independent types of measurements.

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