Datum Deficiency in VLBI Analysis: Case Study of Session 021020XA

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Abstract. This paper presents the results of a case study analysis applied to the Very Long Baseline Interferometry (VLBI) session 021020XA in terms of datum deficiency. An analysis was performed to determine the number of minimal conditions or constraints necessary to remove the datum deficiency (for various parameter combinations). The respective number of singular values from the normal matrix has been tabulated. Five methods, enabling to remove the rank deficiency of the normal equation system, were tested and compared for the quantitative impact on geodetic parameters. It turns out that the datum definition affects most station coordinates as well as polar motion and universal time, up to the 5 mm level.

Keywords. Reference frames, Very Long Baseline Interferometry (VLBI), Earth rotation, geodetic parameters, datum definition

1 Introduction

We distinguish between conditions and constraints: conditions are fulfilled after the adjustment, while constraints may not (depending on the uncontrollable weighting). To remove the datum defect, many different sets of minimal conditions or constraints are possible for the same problem. The various solutions obtained from different sets of minimal conditions or constraints can be related by similarity transformations (Baarda, 1973; Sillard & Boucher, 2001; Even-Tzur, 2006). The only prerequisite for the minimal condition or constraint matrices is that the rank of the matrix including the design matrix and condition matrix equals the number of unknown parameters. Among all possible minimal condition or constraint solutions, the inner condition solution minimizes the weighted sum of the squares of all estimated parameters and the trace of its cofactor matrix. This paper is a case study for the rank deficiency problem for the VLBI session 021020XA observed during the CONT02 campaign, starting at 18h Coordinated Universal Time (UTC) on October 20^{th} 2002 for nearly 24 hours. The International VLBI Service for Geodesy and Astrometry (IVS) scheduled and led the campaign (Schlueter et al., 2002).

In the past, the no-net-rotation condition NNR1 imposed on corrections of quasar positions was applied for retrieving corrections to quasar positions along with corrections to the deficient a priori precession/nutation model, while the no-net-rotation condition NNR2 was applied to the corrections of a priori station coordinates in order to obtain corrections to the latter along with corrections to the a priori polar motion and universal time parameters. The need and use of the no-net-translation (NNT) on corrections of (a set of) a priori station coordinates is unquestionable. Various implementations of the datum definition were studied w.r.t. their influence on the geodetic parameters, i.e., station coordinates, (five) Earth orientation parameters and wet zenith delays (WZD). Physical parameters should be independent of the choice of the datum.

2 Theoretical background

According to geodetic usage, the datum defining condition matrix $\mathbf{B}$ for the parameter corrections $\boldsymbol{\Delta}\mathbf{x}$ reads:

$$\mathbf{B} \cdot \boldsymbol{\Delta}\mathbf{x} = \mathbf{0} \tag{1}$$

After the successful adjustment, equation (1) must be fulfilled up to numerical accuracy.

2.1 Celestial datum points

For celestial datum points, the NNR1 condition matrix $\mathbf{B}_{\mathrm{cel,NNR1,j}}$ is composed (for quasar j) as, e.g., by Kutterer (2004):

$$\mathbf{B}_{\mathrm{cel,NNR1,j}} = \begin{bmatrix} \cos\alpha_j \sin\delta_j & -\sin\alpha_j \\ \sin\alpha_j \sin\delta_j & \cos\alpha_j \\ \cos\delta_j & 0 \end{bmatrix} \quad (2)$$

The meaning of equation (2) is that no global rotational mode is allowed w.r.t. the a priori quasar positions, which are given in a specific celestial reference frame.

2.2 Terrestrial datum points

For terrestrial datum points, the NNT condition matrix $\mathbf{B}_{\mathrm{ter,NNT,i}}$ ensures that the origin of the a priori station network (for station i), given in a clearly predefined terrestrial reference frame, equals the origin of the a posteriori network:

$$\mathbf{B}_{\mathrm{ter,NNT,i}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Equation (3) can also be fulfilled by fixing one single station, however, with the drawback that this station will lack any information concerning precision.

Apart from the NNT condition, the NNR2 condition matrix $\mathbf{B}_{\mathrm{ter,NNR2,i}}$ enforces the orientation of the adjusted station network to be identical to the one of the a priori network:

$$\mathbf{B}_{\mathrm{ter,NNR2,i}} = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ y_i & x_i & 0 \end{bmatrix} \quad (4)$$

A recent review concerning the terrestrial reference frame datum definition is given by Sillard & Boucher (2001).

3 Analysis setup

For the retrieval of the design and weight matrices as well as the reduced observation vector, we used the VLBI software package OCCAM 6.1E (Titov et al., 2004), available at `http://www.hg.tuwien.ac.at/~vlbi`. In addition, the adjusted unknown parameters, as well as their formal errors, served as reference for comparison purposes. The software MATLAB 7.0 was used for testing the impact of the datum on geodetic parameters, i.e., dealing with the datum information w.r.t. the normal matrix.

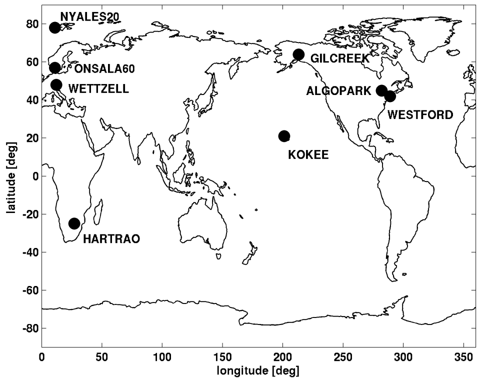

Figure 1. Station network constellation during session 021020XA showing the eight sites.

A short description of the parameterization follows. Station coordinates were estimated once per session for each of the eight sites shown in figure 1. Wet zenith delays were estimated as one offset and hourly piecewise linear functions (PLF) for each of the eight stations. Troposphere gradients were not estimated in order to preserve the present consistent usage of a priori quasar positions and Earth orientation parameters: otherwise the signal of the deficient a priori quasar position model would propagate into the Earth orientation parameters (MacMillan & Ma, 1997). The Earth orientation parameters were parameterized as two nutation offsets (nutation in longitude and in obliquity), two polar motion offsets (x-pole and y-pole), one dUT1 offset and hourly dUT1 rates as PLF. Hourly clock offsets, one 24-hour clock rate and square term were estimated for seven stations. We used hourly clock offsets instead of hourly PLF clock rates because the latter setup is not possible when no constraints are applied. No single loose constraint was used with the exception of defining one condition: the clock of station GILCREEK was fixed as reference. The choice of using GILCREEK instead of, e.g., WETTZELL was motivated by a strong gradient in the clock of the latter. Finally, the elevation cutoff angle was set to 5 degrees. The following a priori models were chosen:

- Terrestrial reference frame (Altamimi et al., 2002) is given by the file ITRF2005.CA1 and includes the source positions according to the ICRF-Ext.2 catalogue (Fey et al., 2004)

- Nutation model IAU2000A (Mathews et al., 2002)

- IERS C04 polar motion and dUT1 (consistent with ITRF2005)
- Vienna Mapping Functions 1 (Boehm et al., 2006).

The following reduction models were applied:

- Solid Earth tides (McCarthy & Petit, 2004)
- Pole tide (Wahr, 1985; McCarthy & Petit, 2004)
- Ocean tide loading (FES2004)
- Atmospheric pressure loading (Petrov & Boy, 2004)
- Zonal tides on dUT1 (Defraigne & Smits, 1999)
- Diurnal/semidiurnal ocean tidal effects on polar motion and dUT1 (Ray et al., 1994; McCarthy & Petit, 2004)
- IERS 92 relativistic consensus model (McCarthy & Petit, 2004).

4 Rank deficiency in VLBI analysis

VLBI is a differential technique, sensitive to relative translations and rotations. Only the use of a priori models together with additional information relevant to the datum allows for recovering conventional geodetic parameters. The number of additional independent information highly depends on the selection of the unknown parameters.

Table 1 shows the number of singular values depending on the selection of estimated parameters. These values are only true if no further singularities appear, e.g., due to the station network geometry or the quasar constellation. In table 1, the station coordinates are always considered as unknowns. The case of solution S8, having 4 singular values, is particularly interesting when deriving dUT1 estimates from intensive sessions. Apart from the NNT condition, it can be seen from equation (4) that one additional NNR2 condition (line 3), which corresponds to a rotation about the a priori z-axis is sufficient to relieve the rank deficiency. Equivalently, fixing the x- or the y-component of one additional station coordinate would be adequate, while fixing the z-component would still lead to a rank deficiency of 1. Each further datum-related information evidently distorts the network geometry. In case of solution S10, where only station coordinate and nutation parameters are estimated, a NNT condition on station coordinate corrections suffices to get rid of the rank deficiency. In solution S1, quasar positions, nutation, polar motion and universal time, and station coordinate corrections were estimated.

Table 1. Rank deficiency depending on selection of unknown parameters. The cross X marks the estimated parameters in the solutions S1 to S10. The abbreviations are: SOL=solution,QUA=quasar positions, NUT=nutation, PM=polar motion, dUT1= universal time, STA=station coordinates, SV=number of singular values (rank deficiency).

SOL	QUA	NUT	PM	dUT1	STA	SV
S1	X	X	X	X	X	9
S2	X		X	X	X	7
S3	X	X			X	6
S4	X			X	X	5
S5	X				X	4
S6		X	X	X	X	6
S7			X	X	X	6
S8				X	X	4
S9					X	3
S10		X			X	3

5 Handling the datum deficiency

In the following, we applied five strategies for handling the datum deficiency, and will shortly describe them by enumeration:

1. M1: using OCCAM 6.1E, which applies NNR2 and NNT to all station coordinate corrections and NNR1 to all quasar position corrections
2. M2: ident to method M1 except that
   - the station coordinate corrections for NYALES20 and HARTRAO were excluded from NNR2 and NNT conditions
   - the five quasar position corrections 1908-201, 1921-293, 2128-123, 0434-188, and 1034-293 were excluded from the NNR1 condition, i.e., those having been observed less than five times during the session (see figure 2)

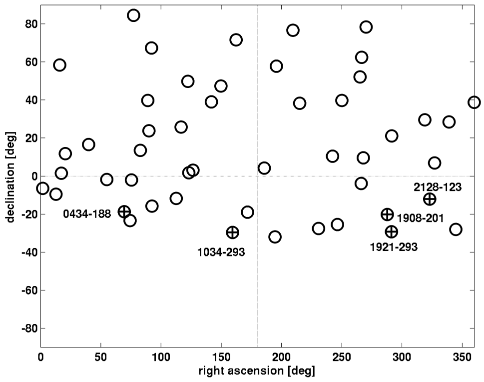

Figure 2. Quasar constellation (49 sources) during session 021020XA. Five sources, removed from NNR1 condition for method M2, are marked by crosses.

3. M3: using the singular value decomposition, following the rank deficiency of table 1

4. M4: using minimal pseudo-observations with robust weighting for station coordinates KOKEE, WESTFORD and the x-component of ONSALA60 as well as for the source position 4C39.25 and the right ascension of source 1357+769

5. M5: ident to method M4, but fixing minimal components, i.e., using minimal conditions instead of constraints.

6 Results

In the following, all results will be shown w.r.t. the ones obtained by method M1 for solution S6, having a rank deficiency of six. Estimating source positions from one single session causes instabilities in, e.g., nutation parameters with the formal errors being multiplied by an order of magnitude. However, in subsections 6.1 and 6.6 the case of solution S1 has been considered, including the quasar positions, and leading to a rank deficiency of nine. The standard deviation of unity weight is 1.24 cm for solution S1, while it is 1.33 cm for solution S6. The improvement of the S1 solution w.r.t. S6 is only about 7% although the $98 = 2 \cdot 49$ additional unknown source position coordinates were estimated. In the S1 solution, station position corrections reach up to 4 cm.

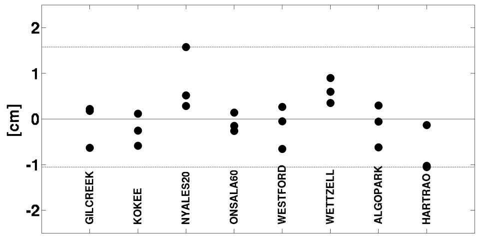

Figure 3. Station coordinate corrections obtained by method M1, solution S6. The three filled circles inherent to one station represent the three geocentric coordinate components x, y, and z.

6.1 Quasar positions

Forty-nine (49) sources have been observed during session 021020XA. Five sources, marked by crosses, have been observed less than five times and were removed from the NNR1 condition for the method M2 (see figure 2). No source with a declination inferior to -40 degrees has been observed.

6.2 Station coordinates

Eight stations observed during session 021020XA (see figure 1). The station coordinate corrections obtained by method M1 for solution S6 are largest (> 1 cm) at the stations NYALES20 and HARTRAO (see figure 3). Both stations were removed from the NNT and NNR2 conditions for the M2 method. These stations were also avoided in the methods M4 and M5. Figure 4 shows the difference in the x-component station coordinates w.r.t. the results obtained by method M1. Differences up to 5 mm do appear w.r.t. method M1, but results do also vary up to 3 mm within the methods M2 to M5. Insufficient modelled or unpredictable station coordinate deformations at specific sites, e.g., due to the incompletely modelled non-tidal ocean/atmoshpere loading effects or displacements induced by minor successive earthquake swarms, do impact the remaining station coordinates at the sub-cm level.

6.3 Nutation, polar motion and universal time corrections

The separation of nutation and polar motion in Very Long Baseline Interferometry (VLBI) is per convention purely frequency-dependent. As we

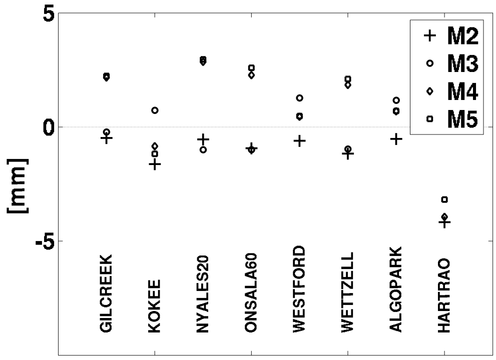

Figure 4. Difference in x-component station coordinates w.r.t. method M1, solution S6.

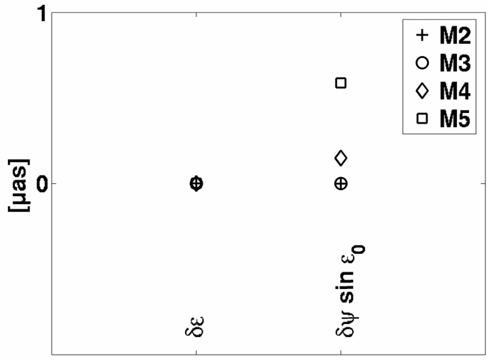

Figure 5. Difference in nutation components w.r.t. method M1, solution S6.

reduced all known diurnal and semi-diurnal effects on polar motion and universal time, we decided to only estimate daily nutation and polar motion parameters. The hourly dUT1 rates by PLF remain greatly unaffected by the choice of parameterization of polar motion and nutation. The differences in nutation (in obliquity $\delta\epsilon$ and longitude $\delta\psi \cdot \sin\epsilon_0$) w.r.t. method M1 are less than 1 μas (see figure 5). The results obtained by methods M1 to M5 are extremely stable for the nutation offset in obliquity $\delta\epsilon$, while the influence of the datum handling is most pronounced for the nutation offset in longitude $\delta\psi \cdot \sin\epsilon_0$. In contrast, the differences in polar motion and universal time between methods M1 and M5 may arise up to 200 μas, which corresponds roughly to 6 mm when projected to Earth surface (see figure 6). Universal time is also given in units of μas for better comparison to polar motion. The x-pole component in polar motion shows the largest variability. Especially the results of method M3 (using the singular value decomposition) differ considerably and systematically w.r.t. method M1.

6.4 Wet zenith delays

We picked out the differences in WZD-rates for station KOKEE w.r.t. method M1 (see figure 7). The x-ticks on the x-axis represent the hourly marks. As expected, WZD should not be affected by how the datum is handled. The differences, which were largest for the station at KOKEE, are less than 4 mm/day, i.e., less than 0.2 mm/hour. The largest variability arises when method M5 is

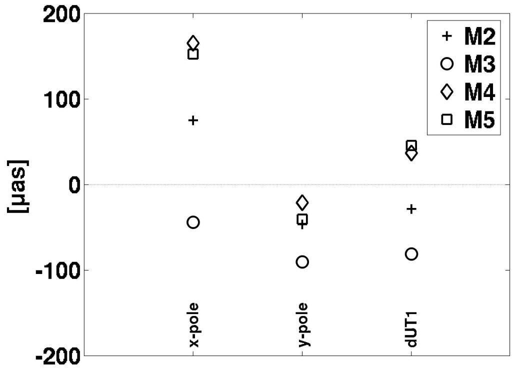

Figure 6. Difference in polar motion and dUT1 w.r.t. method M1, solution S6.

applied. Only methods M1, M2 and M3 provide identical results up to numerical accuracy.

6.5 Clock offsets

The largest differences also arise at station KOKEE for hourly clock offsets (see figure 8). Again, the largest variability (0.5 mm) is obtained when method M5 is used. Here too, only methods M1, M2 and M3 provide stable and identical results.

6.6 Correlation matrix

Figure 9 shows the correlation matrix obtained for solution S1 by method M2. High positive correlations (in white) are visible in hourly clock off-

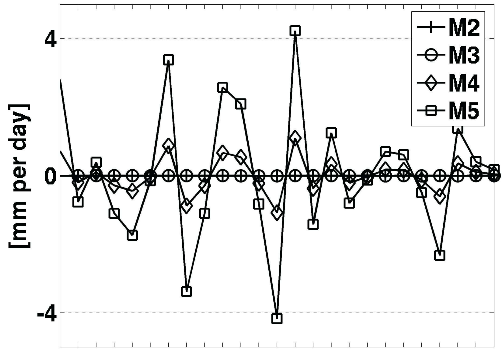

Figure 7. Difference of hourly WZD-rates for station KOKEE w.r.t. method M1, solution S6.

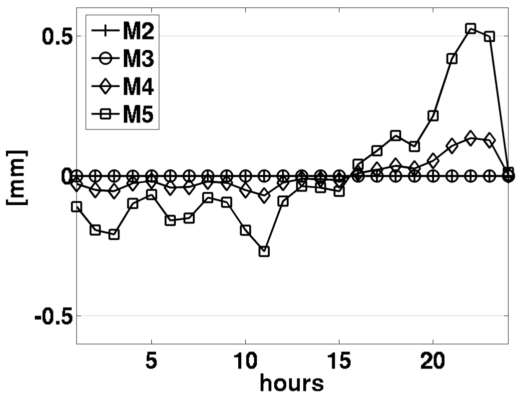

Figure 8. Differences of hourly clock offsets for station KOKEE w.r.t. method M1, solution S6.

sets for individual stations primarily because of the daily estimated rate and square terms in the clocks parameterization, but also due to the presence of station coordinates. If only hourly clock offsets are estimated, then the correlation coefficients between those hourly parameters for each single station will be zero. A cross-correlation of these parameters between stations does of course exist. Non-zero correlation coefficients, in hourly clock offsets within individual stations, are a clear indication of a deficiency in available observations to separate these parameters from station coordinates, especially heights.

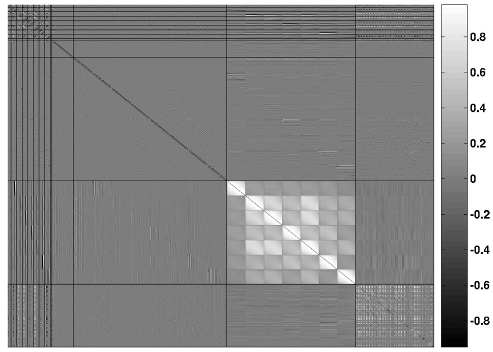

Figure 9. Correlation matrix obtained from solution S1 by method M2.

7 Conclusions

In VLBI analysis, station coordinates, polar motion and universal time corrections are principally affected by the way the datum definition is applied. This is especially true, if displacements in station coordinates arise due to, e.g., the discarding of non-tidal ocean/atmospheric loading effects or minor earthquakes. Auxiliary parameters (WZD and clock offsets) remain highly unaffected by the choice of the datum definition, especially if the datum is handled following the methods M1 to M3.

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