

# **Integration of space geodetic observations and geophysical models**

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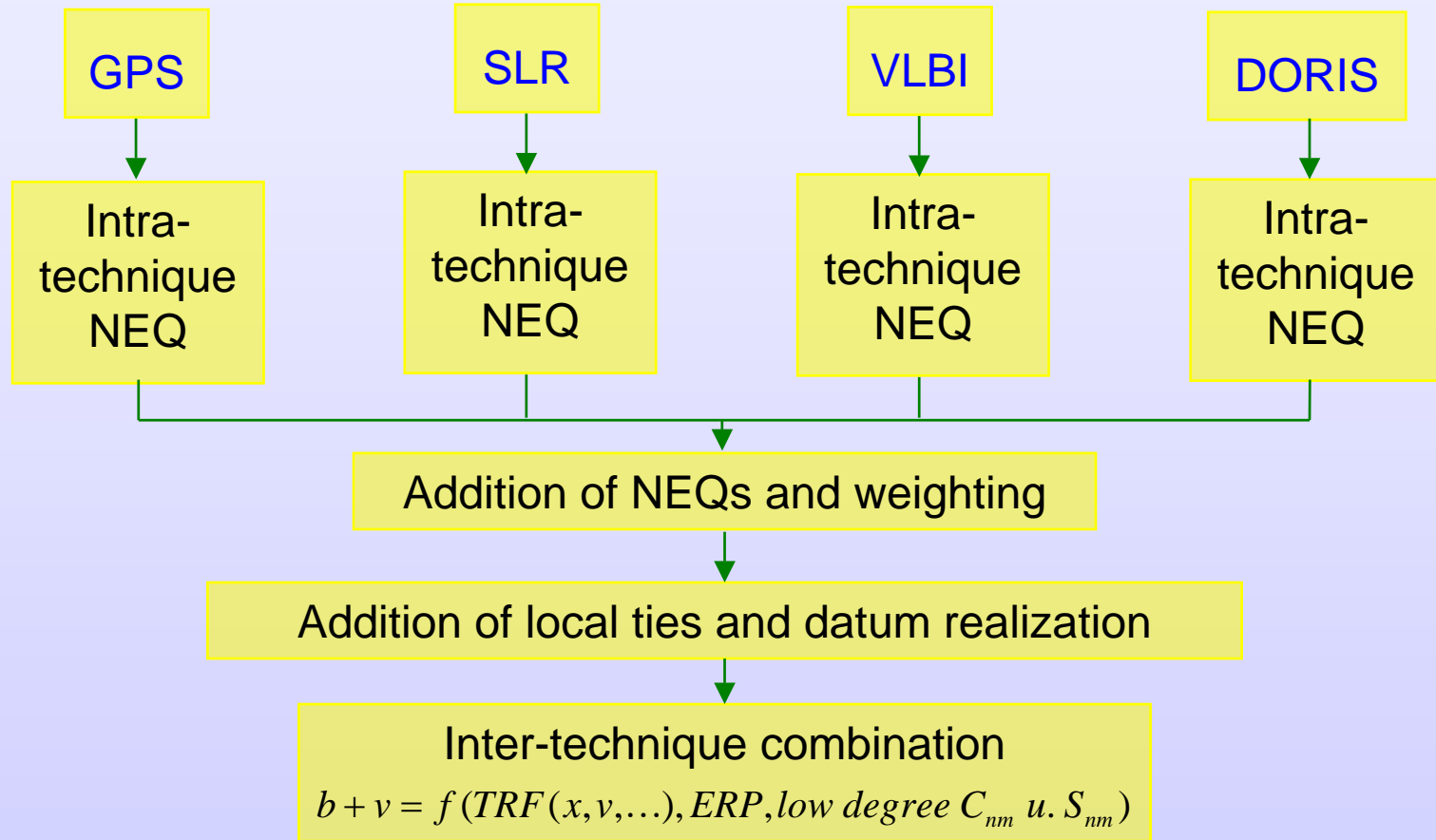
**Geodetic Week 2006, Munich, 12. October 2006**

# Overview

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- Combination of **space geodetic observations** and **geophysical models**
- **Interactions** between the subsystems of the Earth + **mass conservation**
- **Adapting** of geophysical models to gravimetric observations
- Outlook

# Geometric observations



**Site displacements influence the estimation of the “observed” ERP  
⇒ “observed” ERP reflect the rotation of the observation-network!**

# Geophy. models / gravimetric observations

GRACE → gravity field changes → reflect mass redistributions in the Earth

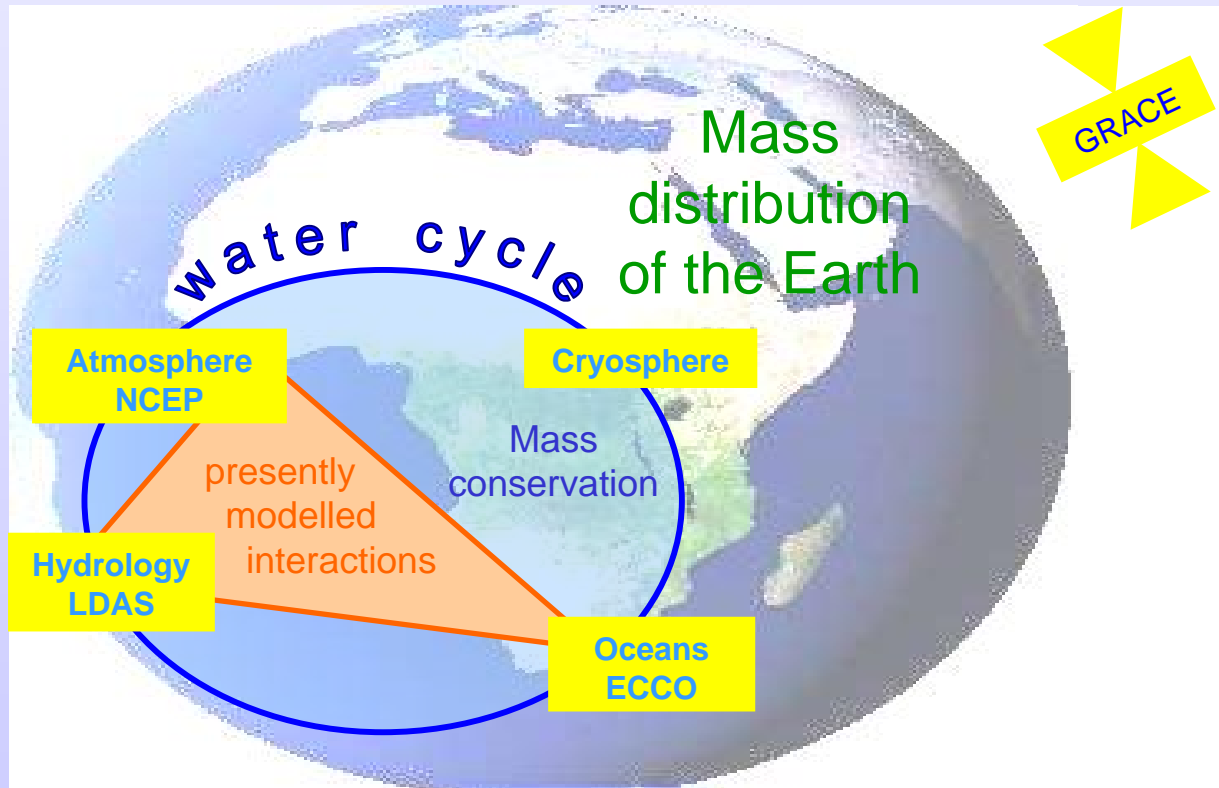
Atmosphere

Oceans

Hydrology

Cryosphere

Mantle, Core



# NEQ-System of the geophysical models

Mass redistribution in the subsystems of the Earth

Atmosphere

Oceans

Hydrology

Cryosphere

Mantle, Core

Excitation functions of the subsystems

$$\chi_i = \chi_i^{matter} + \chi_i^{motion}$$

Conditions for a **consistent combination of the excitation functions** are the description of the interactions and the fulfillment of mass conservation

$$\chi = \sum_{i=1}^n \chi_i \quad \text{equivalent to quasi-observation}$$

Liouville equation is equivalent to quasi-observation-equation

$$\chi + v = f(x_{Pol}, y_{Pol}, \dot{x}_{Pol}, \dot{y}_{Pol}, \Delta LOD)$$

**Mass displacements and motions influence the estimation of the “modelled” ERP  $\Rightarrow$  “modelled” ERP reflect the rotation of the Earth!**

# Combination of all “observations”

Inter-technique combination of the  
geometric observations  
NEQ

Combination of the  
geophysical quasi-observations  
NEQ

Addition of NEQs and **weighting**

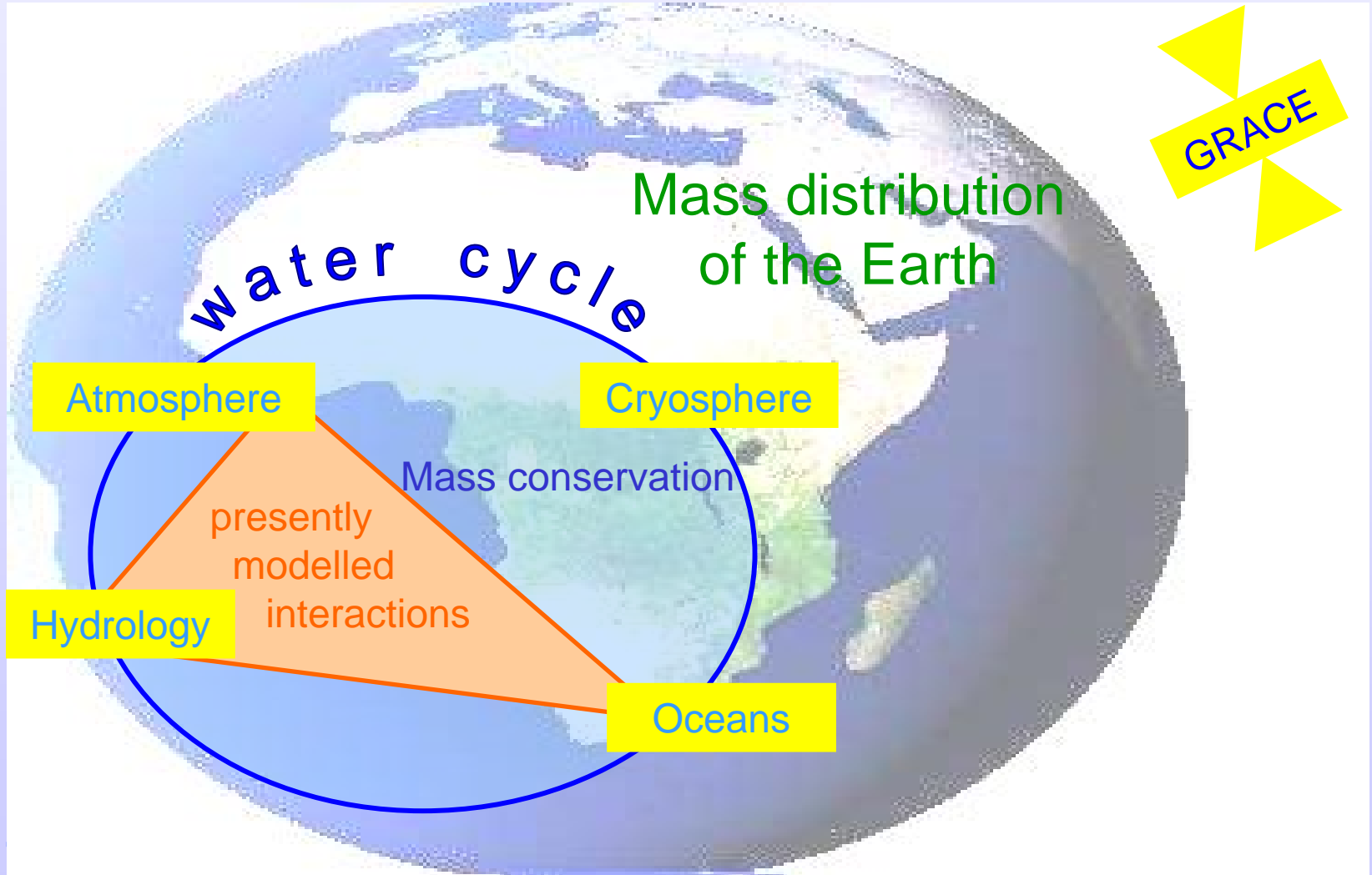
Datum realization

Inter-technique combination of all “observations”

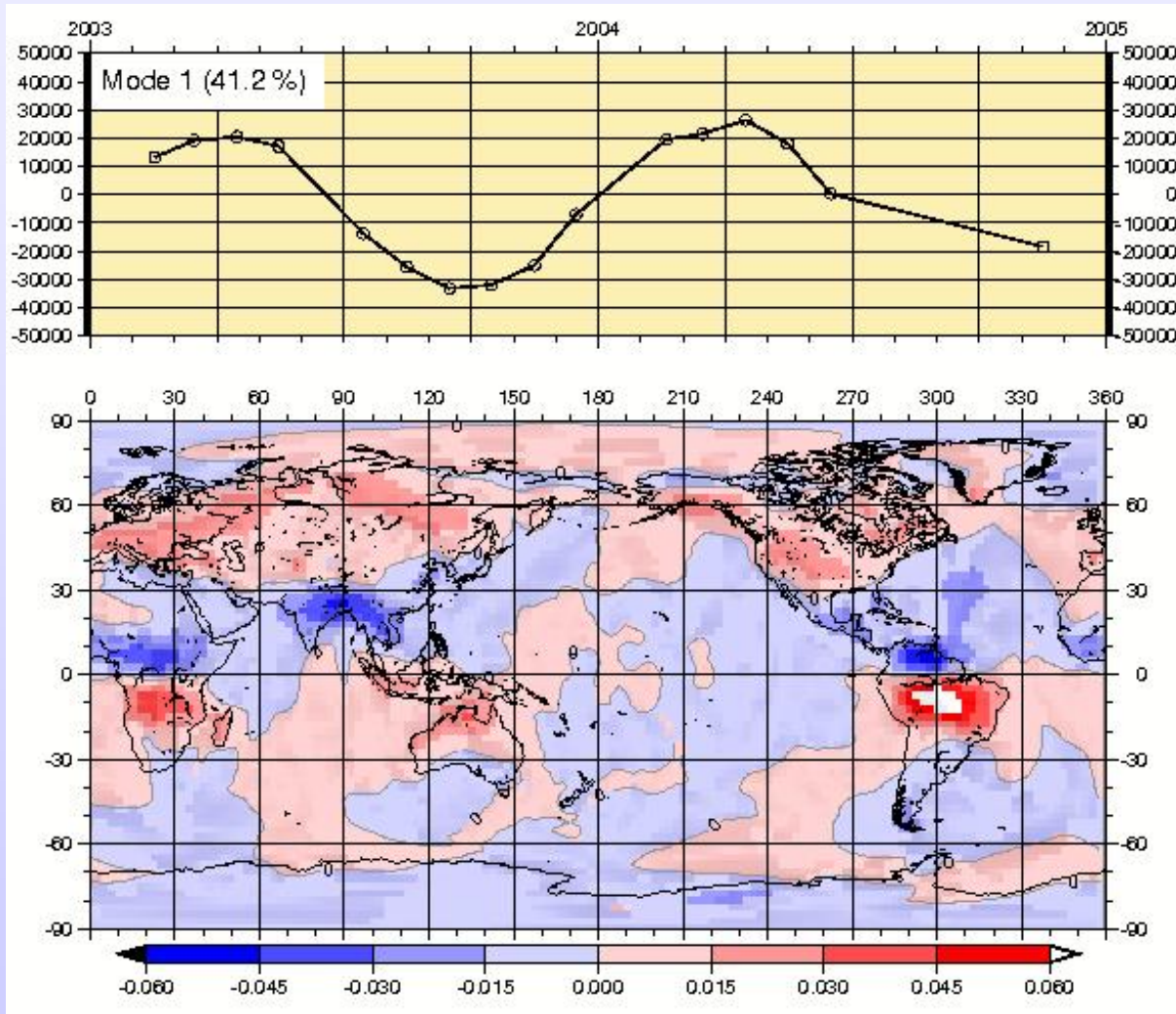
$$b + v = f(TRF(x, v, \dots), ERP, \text{low degree } C_{nm} \text{ u. } S_{nm})$$

**Network uncertainties do not appear in the ERP, but could under circumstances identified as „outliers“ in the TRF!**

# Improvement of geophysical models



# Principal Component Analysis PCA



**Principal components:**  
Represent the temporal evolution of the intensity



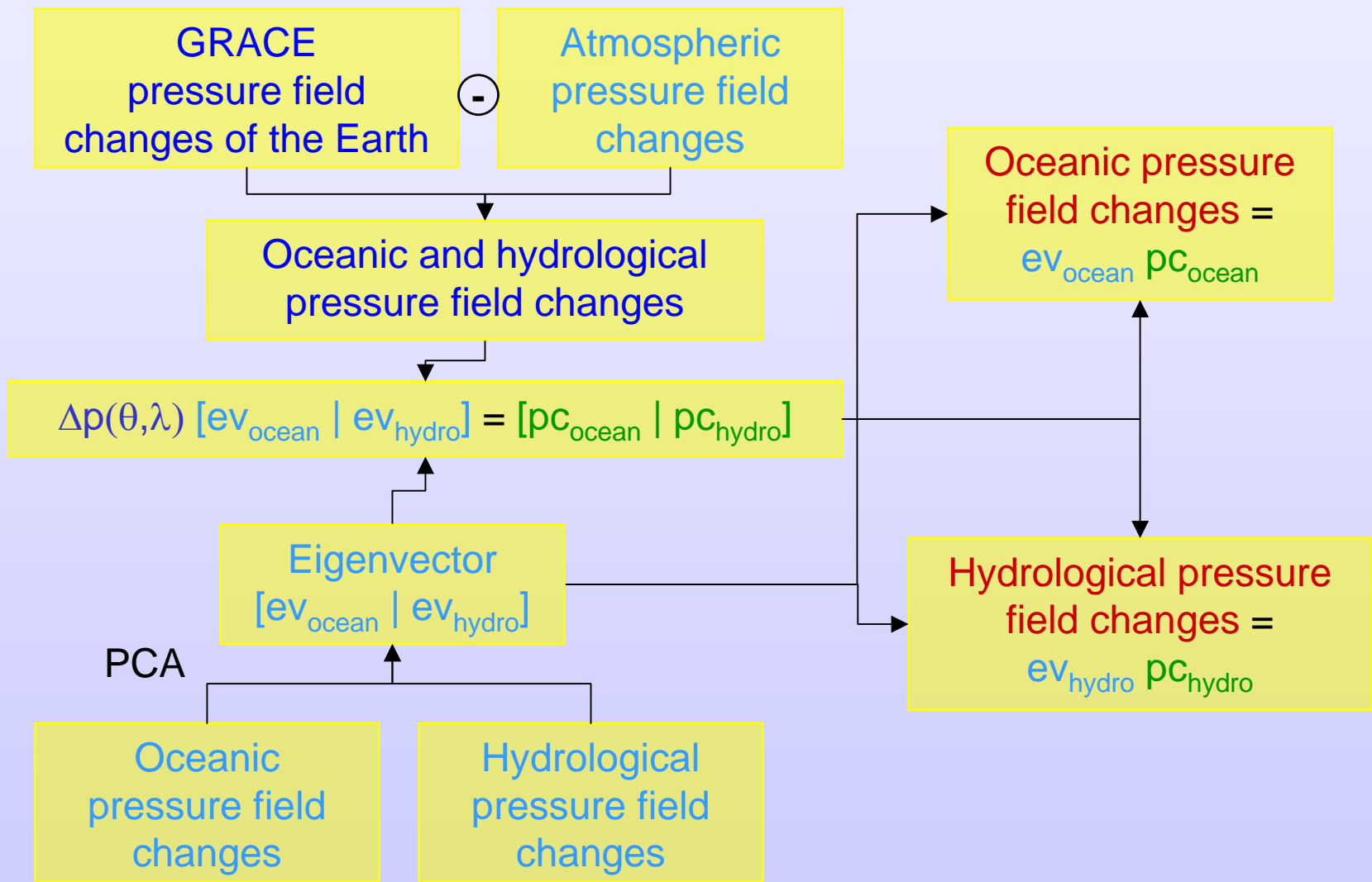
**Eigenvector:**  
Represents dominant spatial patterns of variability



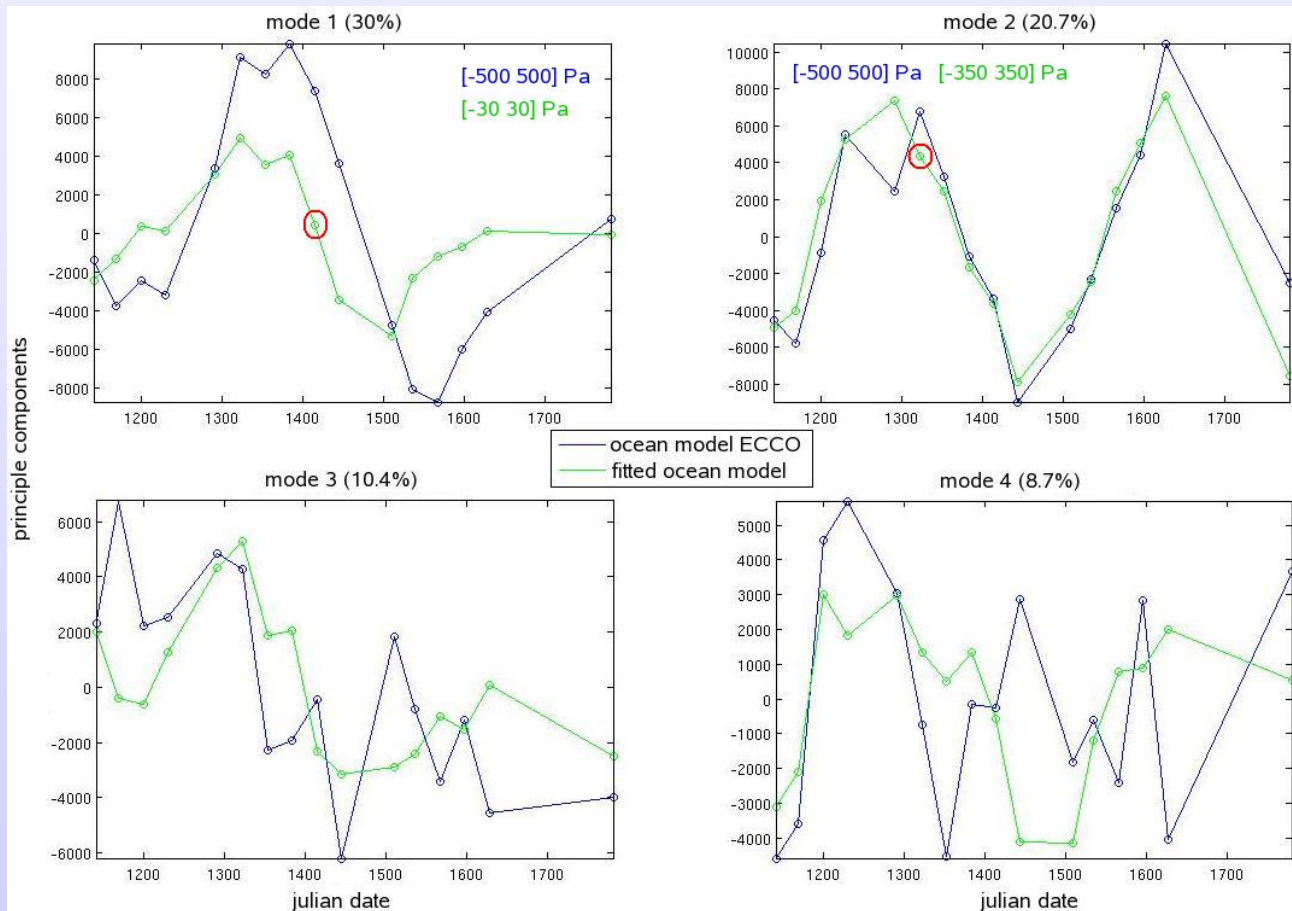
**Signal:**  
Represents spatial and temporal patterns



# Adapting of geophy. models to GRACE-obs.

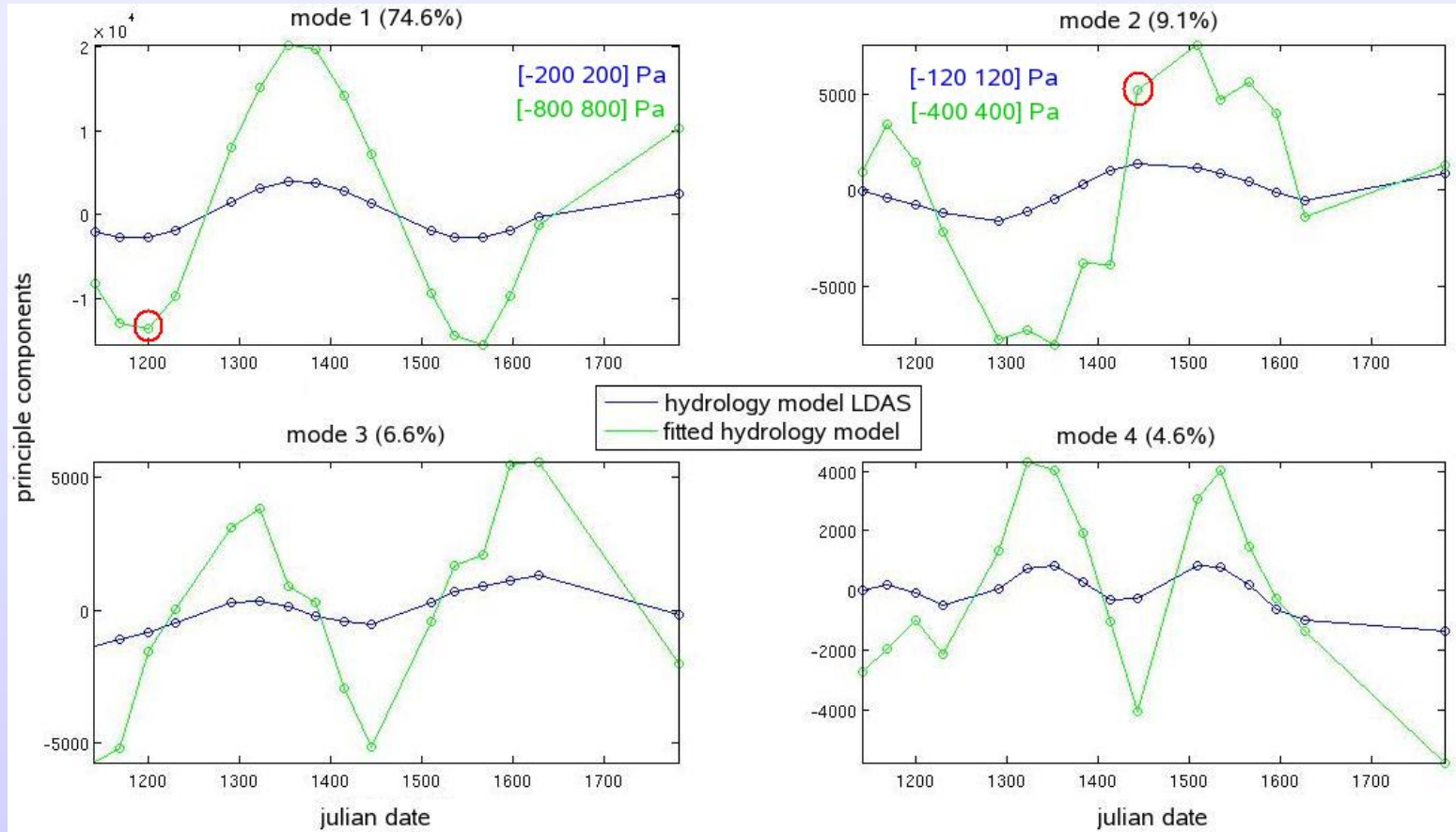


# Comparison of principal components



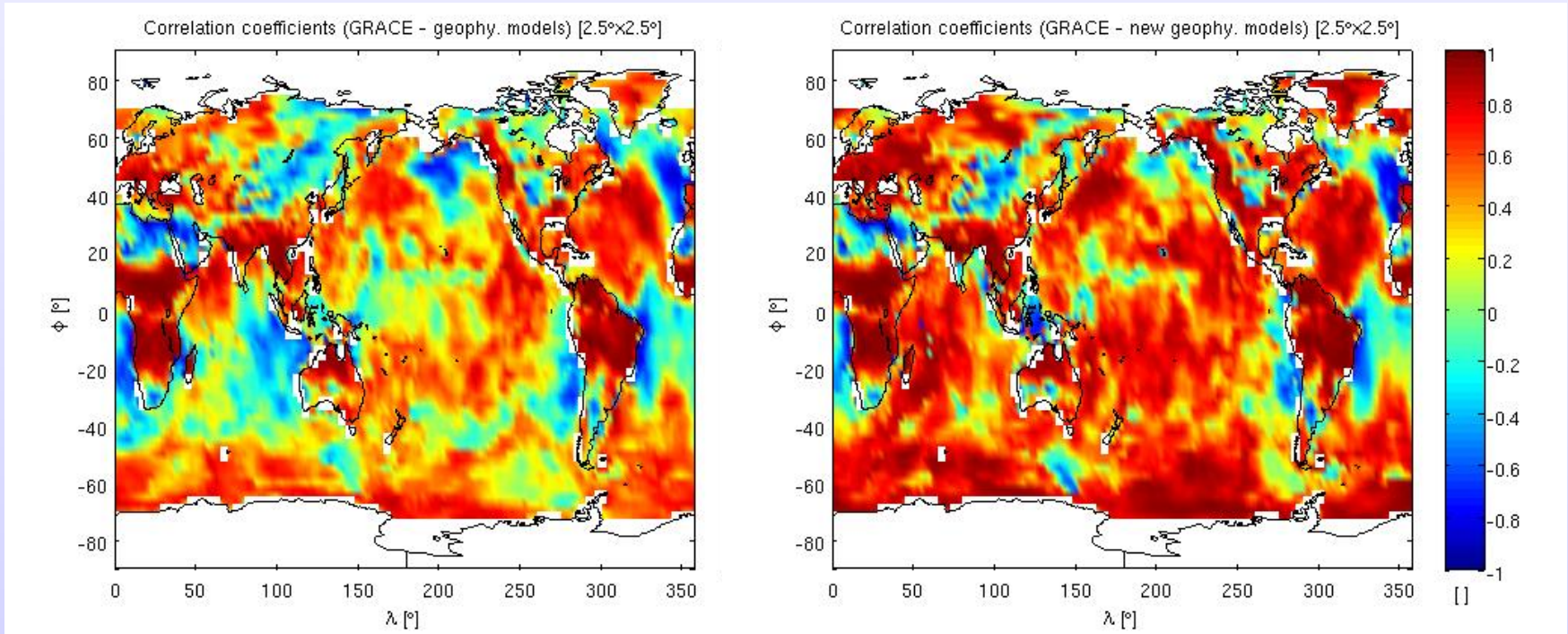
**Pressure field changes of the ocean model ECCO are smaller than of the adapted ocean model!**

# Comparison of principal components



**Pressure field changes of the hydrology model LDAS are much smaller and smoother than of the adapted hydrology model!**

# Comparison of correlation-coefficients



**mean value = 0.2871**  
**standard deviation = 0.4046**

**mean value = 0.4751**  
**standard deviation = 0.4105**

# Outlook

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- Computation of the **excitation functions** (matter-term)
- **Combination** of the “modelled” excitation functions of all relevant subsystems
- **Comparision** with the “observed” excitation function
- Estimation of the **accuracy** of the “modelled” excitation functions

**Thank you  
for your attention!**

# GRACE data processing

**Level 2 product GSM**  
monthly static gravity field  
exclusive non-tidal atmospheric  
and oceanic gravity field part

**Level 2 product GAC**  
monthly non-tidal atmospheric  
and oceanic static gravity field

$$\Delta C_{nm} S_{nm} = (GSM - \overline{GSM}) + (GAC - \overline{GAC})$$

Correlated error filter form DON CHAMBERS

Gaussian filter ( $r = 500\text{km}$ ) form JOHN WAHR

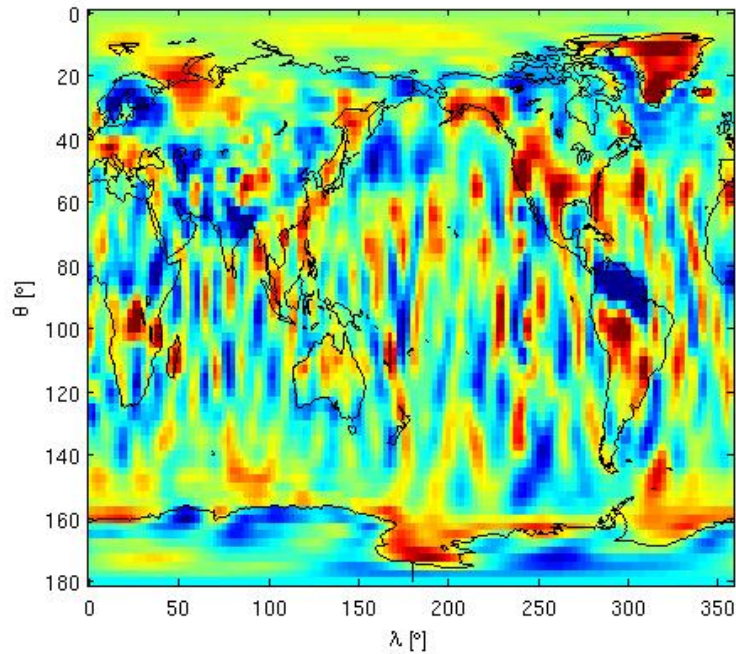
**Pressure field changes of the Earth**

$$\Delta p(\theta, \lambda) = \frac{a_E g \rho_E}{3} \sum_{n=0}^{60} \sum_{m=0}^n \frac{2n+1}{1+k_n} \overline{P}_{nm}(\cos \theta) [\Delta C_{nm} \cos(m\lambda) + \Delta S_{nm} \sin(m\lambda)]$$

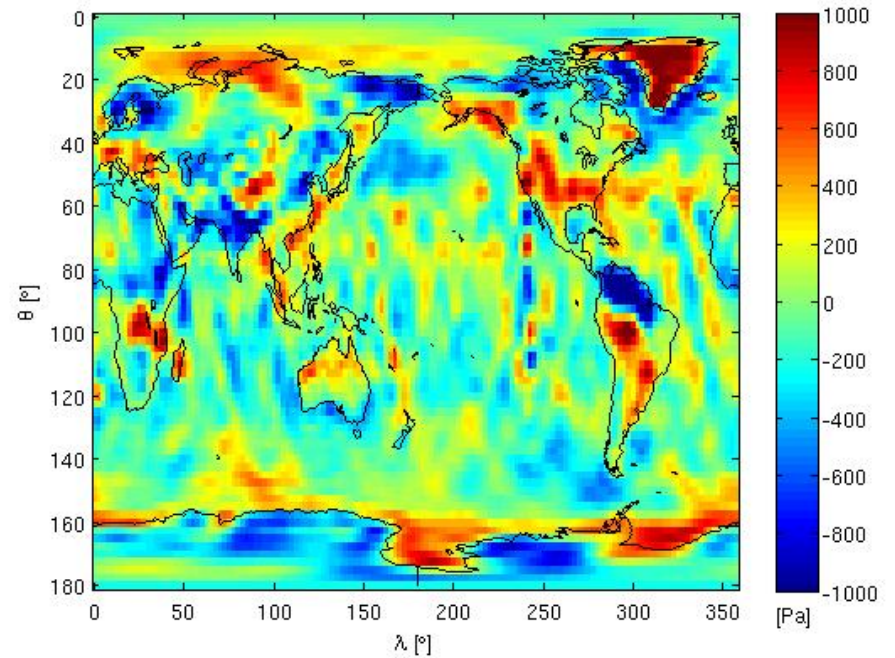


# Pressure field changes

Gaussian filtered



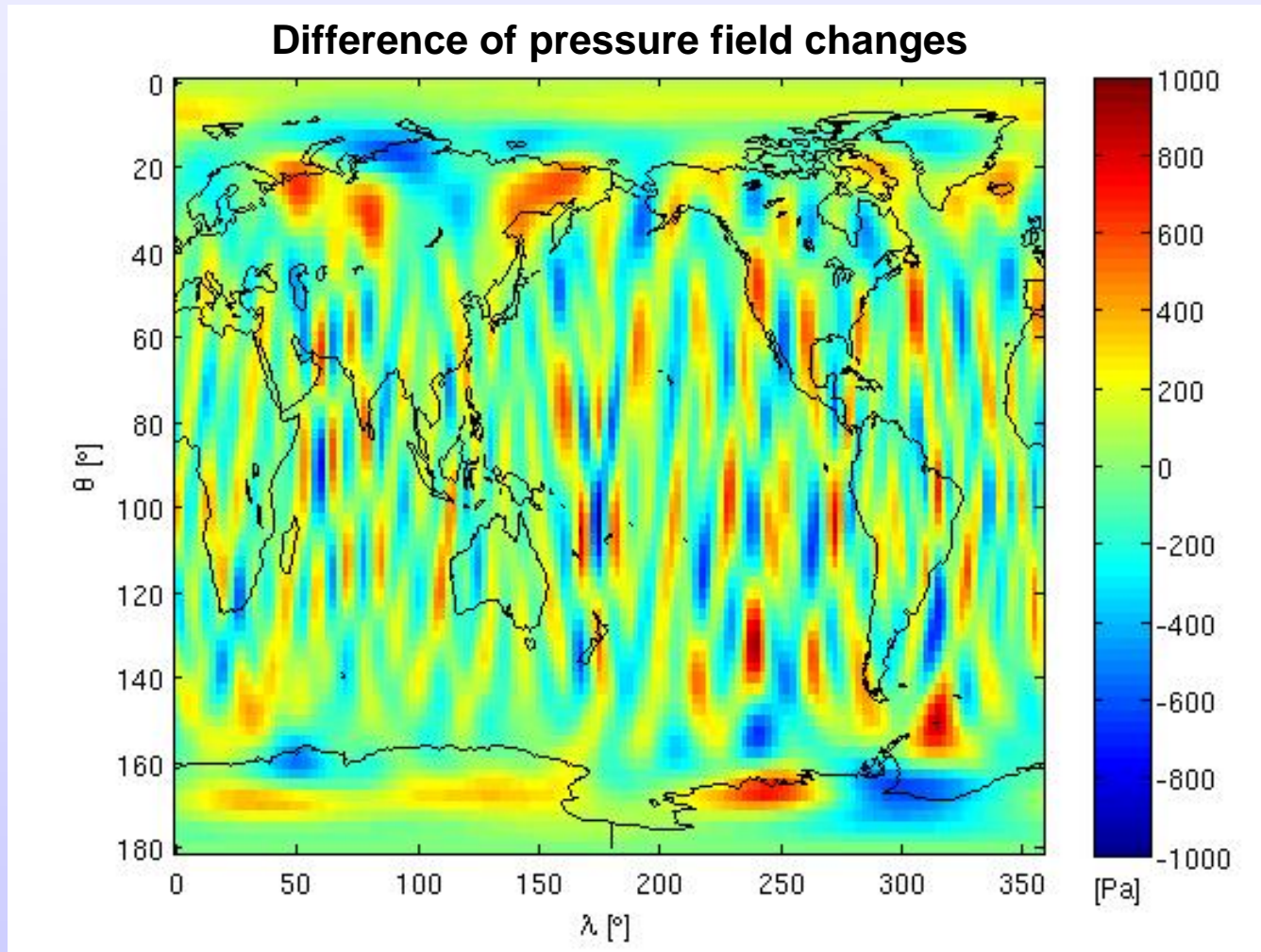
Correlated error and Gaussian filtered



**N-S-stripes disappear!**



# Difference of pressure field changes



# Calculation of excitation function

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$$\chi_1^{matter} = -\frac{a^2}{(C-A)\left(1-\frac{k_2}{k_f}\right)} \int_{\lambda} \int_{\varphi} \rho \xi \cos \varphi \sin \varphi \cos \lambda a^2 \cos \varphi d\varphi d\lambda$$

$$\chi_2^{matter} = -\frac{a^2}{(C-A)\left(1-\frac{k_2}{k_f}\right)} \int_{\lambda} \int_{\varphi} \rho \xi \cos \varphi \sin \varphi \sin \lambda a^2 \cos \varphi d\varphi d\lambda$$

$$\chi_3^{matter} = -\frac{a^2}{C_m} \int_{\lambda} \int_{\varphi} \rho \xi \cos^2 \varphi a^2 \cos \varphi d\varphi d\lambda$$

# Calculation of excitation function

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$$\chi_1^{matter} = -\frac{M_E a^2}{(C - A) \left(1 - \frac{k_2}{k_f}\right)} \Delta C_{21}$$

$$\chi_2^{matter} = -\frac{M_E a^2}{(C - A) \left(1 - \frac{k_2}{k_f}\right)} \Delta S_{21}$$

$$\chi_3^{matter} = -\frac{2 M_E a^2}{3 C_m} \Delta C_{20}$$

# Calculation of excitation function

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$$\chi_1^{motion} = -\frac{1}{\Omega(C-A)\left(1-\frac{k_2}{k_f}\right)} \int \int \int \rho r (u \sin \varphi \cos \lambda - v \sin \lambda) r^2 \cos \varphi d\varphi d\lambda dr$$

$$\chi_2^{motion} = -\frac{1}{\Omega(C-A)\left(1-\frac{k_2}{k_f}\right)} \int \int \int \rho r (u \sin \varphi \sin \lambda - v \cos \lambda) r^2 \cos \varphi d\varphi d\lambda dr$$

$$\chi_3^{motion} = -\frac{1}{\Omega C_m} \int \int \int \rho r^2 \cos \varphi d\varphi d\lambda dr$$

# NEQs

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$$\chi_1(t) = x(t) + \frac{1}{\frac{2\pi}{T_c} \left(1 + \frac{1}{4Q^2}\right)} \left( \frac{1}{2Q} \dot{x}(t) + \dot{y}(t) \right)$$

$$\chi_2(t) = -y(t) + \frac{1}{\frac{2\pi}{T_c} \left(1 + \frac{1}{4Q^2}\right)} \left( \dot{x}(t) - \frac{1}{2Q} \dot{y}(t) \right)$$

$$\chi_3(t) = -\frac{\Delta LOD(t)}{LOD}$$

# Correlated error filter

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- N-S-strips are a result of systematic errors in the coefficients.
- Coefficients up to order and degree 7 do not show syst. errors.
- For higher orders the systematic errors are estimated by fitting polynomials  $p(x)$  of degree 7 by least square adjustment.

**Observation equation:**  $p(x) = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + C$

**Unknowns:**  $a, b, c, d, e, f, g, C$

**Observations:**  $C_{9,8}, C_{11,8}, C_{13,8}, \dots, C_{81,8}$

**Filtered coefficients:**  $C_{n,8}^{\text{filtered}} = C_{n,8} - p(n)$

- To estimate the fitting polynomials for orders higher than 40 you have to use coefficients with degrees up to 40+order.
- Coefficients higher than order and degree 50 are not filtered by the correlated error filter.

# Isotropic Gaussian filter

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$$W_n = \exp[-(n r_s/a_E)^2/(4 \ln 2)]$$